An Empirical Test of Mean-Variance Optimization

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The past 15 years have seen Markowitz Mean-Variance Optimization grow from an obscure academic argument for diversification to a ubiquitous elixir to cure what ails the portfolio of every individual investor. This study tests how using Mean-Variance Optimization to reposition a three-asset portfolio on an annual basis would have faired against various fixed allocations with equivalent expected risk. Optimizations were performed using six different methods to derive inputs and over three different look-back periods. Results indication that most input derivations methods cause the ":optimized" portfolios to underperform their fixed allocations counterparts.

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"Markowitz (1959) mean-variance (MV) efficiency is the classic paradigm of modern finance for efficiently allocating capital among risky assets. Given estimates of expected return, standard deviation or variance, and correlation of return for a set of assets, MV efficiency provides the investor with an exact prescription for optimal allocation of capital. The Markowitz efficient frontier represents all efficient portfolios in the sense that all other portfolios have less expected return for a given level of risk or, equivalently, more risk for a given level of expected return. In this framework, the variance or standard deviation of return defines portfolio risk. MV efficiency considers not only the risk and return of securities but also their interrelationships."

That opening paragraph, from Richard Michaud's 1998 book, *Efficient Asset Management*, seems to be a fitting description of the expectations of many investment consultants when it comes to Mean-Variance Portfolio Optimization (MVPO). Each year the oversight committees of hundreds of portfolios direct their consultants to go through the MVPO exercise to either validate their current asset allocation or prompt them to make changes. But is this sophisticated mathematical paradigm a magic elixir or just so much snake oil?

Versus a Naive Fixed Allocation

If MVPO lives up to its promise, then an optimized portfolio should be able to show consistent value added over a naïve, fixed asset allocation. To explore this assertion, we examined the application of MVPO to maximize the return of a portfolio of three assets on an annual basis. The expected risk, or standard deviation, of the optimized portfolios was constrained to be no greater than that of the Naive Allocation. In addition, optimized allocations were constrained to an 80% maximum and 10% minimum for any single asset class. Naive Allocation standard deviation was calculated using the same inputs as the optimization method being tested. The three assets chosen for this experiment were US equity, International equity, and US fixed income. Each asset is represented by an appropriate index: the Standard & Poors 500 (S&P 500), the Morgan Stanley Capital International Europe Australasia Far East (EAFE), and the Lehman Brothers Aggregate Bond (LB Agg) Indices respectively.[1] The Naive Portfolio allocation was chosen as 35% S&P 500, 35% EAFE, and 30% LB Agg, close to an even split, but also simulating a classic 70% equity, 30% bond allocation.

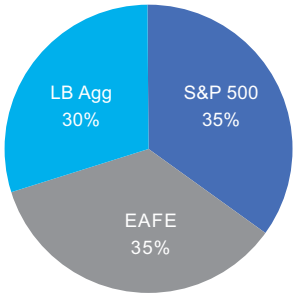


Figure 1 - Naive Allocation

Utilizing index returns from 1970 through 2000, we tested optimized portfolios at the beginning of each year for 1981 through 2001. Quarterly return information was used to calculate the MVPO inputs, and it was assumed that only return data up through the third quarter of the preceding year was known at each optimization. For example, in the fourth quarter of 1980, quarterly returns through September 30, 1980, are used to calculate means, standard deviations, and correlations. These values were then used as inputs for the MVPO that positioned the portfolio for calendar year 1981. No transaction costs were assumed when adjusting the portfolios to the new optimal structure. MVPO was tested using six variations on the derivation of input estimates as described in Table 1. In addition, three different look-back periods were used: 3, 5, and 10 years. Table 2 shows the results of the different MVPO paradigms

[1] Since the Lehman Brothers Aggregate Bond Index began back in 1976, US Bond return data was used to augment to first six years in order to have data back to 1970 for all three assets.

over the 21 years considered in the study. The last column in the table, labeled "Significance," shows the probability that the *actual* average quarterly return premium over the Naive Allocation (third to last column) is actually zero. Charts of the annual allocation changes associated with each MVPO paradigm are located in the appendix.

The first observation from Table 2 is that the majority of the "optimized" paradigms show no significant difference in return compared to the Naive Allocation. The one exception is the FER paradigm, which yields superior results across all but one look-back period. At first glance it appears that the longer the look-back period, the more successful the MVPO paradigm; however, results for the two Modified FER paradigms contradict this conclusion. In the Modified FER paradigms, returns are fixed and standard deviations are derived from 10 year rolling histories. Thus, asset correlations are the only inputs that are derived from the shorter-term 3- and 5-year return histories.

These results suggest that the nature of asset correlations require a shorter, or more change-sensitive, measurement period. An analysis of the behavior of asset correlations lends support to this assertion.

Figure 2 indicates that correlations tend to go through

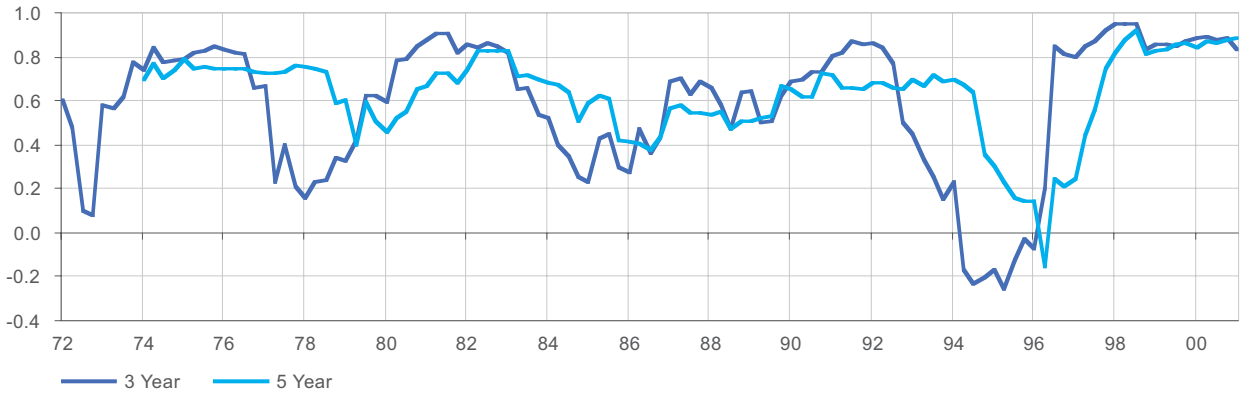
Table 1 - Variations on the derivation of MVPO inputs

Paradigm	Description
Rear-View Mirror (RVM)	All inputs are derived from historical return for the specific look-back period
Fixed Expected Returns (FER)	Return inputs remain the same for all annual optimizations. Expected annual returns were set at 10% for
Long-Term Returns (LTR)	Return inputs consider all available history (from 1970) up to the quarter prior to optimization. Other inputs derived using RVM.
Modified RVM	Standard deviations are set using a 10-year look-back with all other inputs derived as in RVM.
Modified FER	Standard deviations are set using a 10-year look-back with all other inputs derived as in FER.
Modified LTR	Standard deviations are set using a 10-year look-back with all other inputs derived as in LTR.

Table 2 - Returns and Statistics Resulting From Optimizations Using Naive Portfolio Risk

	Annualized Return	Annual Std. Dev.	Sharpe Ratio	Premiums Over Naive Allocation			
Naive Allocation	12.24%	12.03%	0.4642				
10-Year Look-back				Annualized Premium	Quarterly Average	Quarterly Std. Dev.	Significance
Rear-View Mirror	11.98%	11.69%	0.4551	-0.26%	-0.07%	1.47%	66.33%
Fixed Expected Returns	13.33%	11.92%	0.5598	1.09%	0.25%	1.11%	2.01%
Long-Term Returns	12.21%	12.57%	0.4415	-0.04%	-0.01%	1.38%	51.49%
5-Year Look-back							
Rear-View Mirror	11.91%	11.03%	0.4763	-0.33%	-0.08%	1.78%	66.73%
Fixed Expected Returns	12.50%	12.79%	0.4563	0.25%	0.07%	1.14%	29.24%
Long-Term Returns	12.38%	12.85%	0.4451	0.14%	0.04%	1.17%	38.08%
3-Year Look-back							
Rear-View Mirror	12.41%	12.34%	0.4665	0.17%	0.04%	1.88%	41.60%
Fixed Expected Returns	11.62%	13.23%	0.3748	-0.62%	-0.11%	1.62%	74.10%
Long-Term Returns	11.64%	13.23%	0.3768	-0.60%	-0.11%	1.63%	73.25%
Modified 5-Year Look-back (10-Year Standard Deviations)							
Rear-View Mirror	11.90%	10.80%	0.4854	-0.34%	-0.09%	1.74%	68.99%
Fixed Expected Returns	13.29%	11.90%	0.5571	1.04%	0.24%	1.12%	2.56%
Long-Term Returns	12.24%	12.79%	0.4363	0.00%	0.00%	1.37%	49.11%
Modified 3-Year Look-back (10-Year Standard Deviations							
Rear-View Mirror	12.20%	11.36%	0.4875	-0.04%	-0.02%	1.69%	54.91%
Fixed Expected Returns	13.38%	12.05%	0.5578	1.14%	0.26%	1.11%	1.62%
Long-Term Returns	12.26%	12.84%	0.4359	0.01%	0.01%	1.35%	47.02%

Figure 2 - Correlation of Quarterly Returns S&P 500 vs. MSCIEAFE



"phases," shifting from one level to another and persisting at that level for a non-trivial time. For example, in Figure 2 the 3-Year correlation falls to zero at the beginning of 1994 and does not turn positive again until the beginning of 1996, while the 5-Year correlation does not fully detect this shift until it is practically over. Hamilton (1989) introduced this concept as it relates to return variance in his work on modeling investment return distributions.

The fact that the use of historical *returns* as inputs for the MVPO paradigms rendered them ineffective may be surprising to some; however, this result lines up with the findings of Chopra & Ziemba (1993) that errors in expected return inputs are more than ten times more damaging to optimization results than errors in expected standard deviations, and twenty times more damaging than errors in expected covariances. Since these are *estimates* of unknowable parameters for *hypothesized* (and equally unknowable) distributions, errors are a virtual certainty. The FER paradigm reduces this aspect by adopting a neutral return position between the two equity asset classes. Modifications to using historical mean returns as inputs to MVPO have been proposed for years. Michaud (1998) provides a summary of several different approaches. These adjusting algorithms generally result in a reduction in expected return disparities between asset classes, in some cases becoming almost indistinguishable from the neutral position of the FER paradigm.

Versus a Prescient Fixed Allocation

The Naive Allocation in the previous section served as a risk benchmark for the optimized portfolio paradigms that attempted to maximize portfolio return. But it wasn't necessarily the best performing fixed allocation for the 21-year period being considered. How would the MVPO paradigms stack up against the fixed allocation with the highest average return for the21-year period, with overall risk roughly equal to that of the Naive Allocation? Applying the same 10% minimum and 80% maximum allocation constraints as on the optimized portfolios, this fixed allocation – what we have called the Prescient Allocation – was composed of 62% S&P 500, 10% EAFE, and 28% LB Agg (rounding to the nearest whole percent.) The results of the MVPO paradigms are listed in Table 3.

Rerunning the experiment using the annual target risk of the Prescient Allocation, the same general pattern emerged with the Modified 3-Year FER paradigm out-performing by 59 basis points per year. Although this

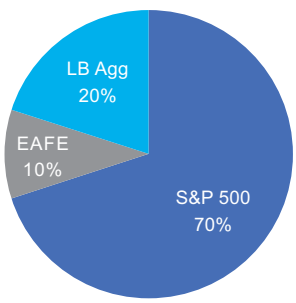


Figure 3 - Prescient Allocation

premium is only statistically significant at the 13% level, the fact that it is generated by the same MVPO paradigm that outperformed in the Naive Portfolio experiment is certainly notable.

These experiments offer compelling evidence that MVPO paradigms using biased return and short-term risk inputs are actually more likely to destroy value than to add value, especially if transaction costs are incorporated. The temptation to use biased estimates for asset returns is intuitively appealing, but immensely destructive. And while the results for the Modified 3-Year FER paradigm are encouraging, they require further examination.

From a practical standpoint, our experiment makes one very big assumption that we have not yet addressed: It is assumed that those in charge of portfolio oversight would follow the MVPO paradigm somewhat blindly and make the appropriate allocation adjustments no matter how seismic the shifts. Clearly, the reticence of a majority of trustees to implement a prescribed allocation shift could easily derail any potential for even the best MVPO paradigm to add value.

To test the impact that Trustee intervention in the MVPO paradigm might have on the above results, the Modified 3-Year FER paradigm was rerun with the

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Long-Term Returns	12.26%	12.84%	0.4359	0.01%	0.01%	1.35%	47.02%

Table 4 - Returns and Statistics Resulting from Optimizations Using 3-Year Look-back Constrained FER Paradigms

				Premiums Over Appropriate Fixed Allocation			
	Annualized Return	Annualized Std. Dev.	Sharpe Ratio	Annualized Premium	Quarterly Average	Quarterly Std. Dev.	Significance
Naive Allocation	12.24%	12.03%	0.4642				
FER (with Naive Risk)	13.42%	11.90%	0.5682	1.18%	0.27%	1.02%	0.85%
Prescient Allocation	13.11%	11.25%	0.5735				
FER (with Prescient Risk)	13.39%	11.67%	0.5768	0.28%	0.08%	0.82%	18.65%

Figure 4 - FER Paradigm Naive Risk Optimizer Shifts

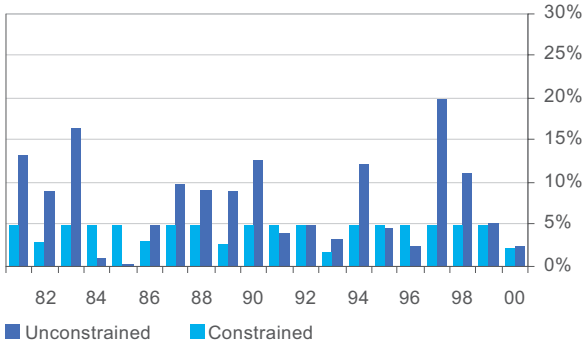
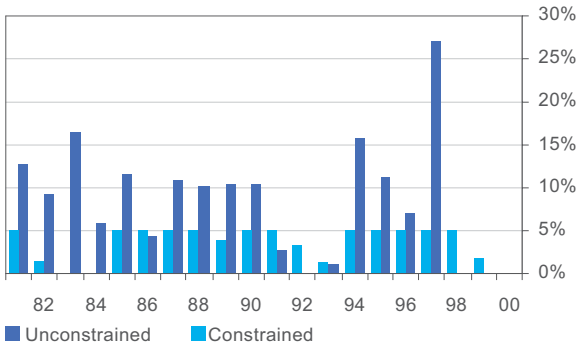


Figure 5 - FER Paradigm Prescient Risk Optimizer Shifts



additional constraint that annual allocation shifts could not exceed 5% of total assets. This necessitated allowing the expected risk of the optimized portfolios to at times exceed the target (i.e., the expected risk of the fixed blend allocation) as the magnitude of some covariance shifts required multiple years to appropriately re-align the optimized portfolio in the prescribed 5% increments. The results of the constrained shift MVPO paradigms are shown in Table 4. Surprisingly, the optimized portfolio corresponding to the Naive Allocation risk levels was actually slightly better than the paradigm with unconstrained shifts (see Table 2). This result did not hold for the Prescient Allocation optimization, which was slightly less effective when shifts were constrained.

Under the Hood

An examination of the inputs to the Modified 3-Year FER is enlightening. Figure 5 shows a chart of the allocation between **equities** of the constrained paradigm overlaid with lines graphing the 3-year correlations of the S&P 500 with the EAFE and the 10-year standard deviations of each index. Since expected returns are equal, expected standard deviations provide the optimizer's bias between the two equity assets, and it will favor the less risky asset (the S&P 500), only increasing allocations to the riskier equity asset if a lower correlation offsets the additional risk. Since EAFE standard deviations are greater throughout the 21-year experiment, the S&P 500 receives the majority allocation. Only at the end of 1983 do the 10-year standard deviations of the two assets converge, resulting in an almost 50/50 split between them (as long as correlation is less than one).

Does allowing asset allocations to be driven by long-term risk and short-term correlations make sense? Clearly the only return prediction in the FER paradigm was that

Figure 6 - Optimizer Equity Inputs and Resulting Equity Allocations for 3-Year Modified FER

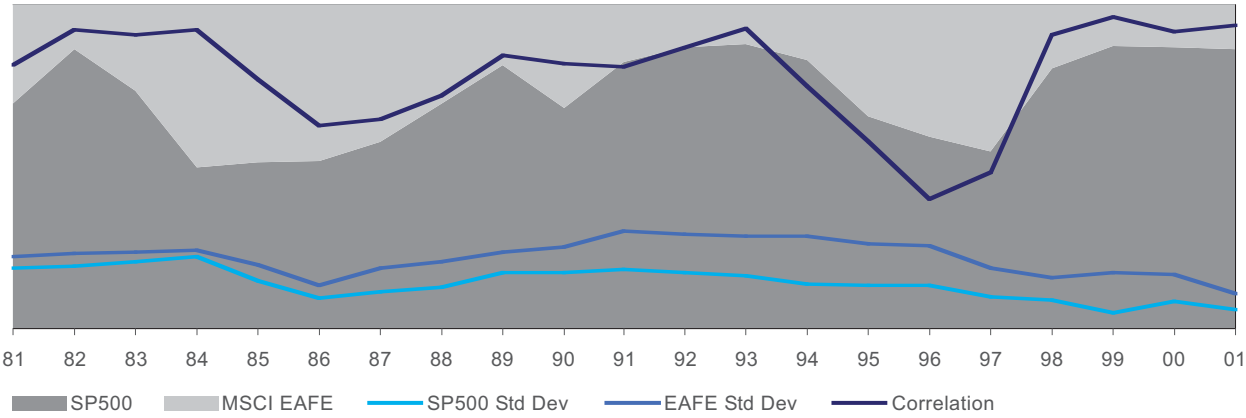


Table 5 - Average Allocation of MVPO Paradigms

	S&P 500	EAFE	LB Agg	Total Equity
Naive Blend	35.0%	35.0%	30.0%	70.0%
10-Year Look-back				
RVM	35.0%	30.2%	34.8%	65.2%
FER	53.1%	19.8%	27.0%	73.0%
LTR	25.4%	40.0%	34.6%	65.4%
5-Year Look-back				
RVM	42.3%	23.0%	34.7%	65.3%
FER	41.1%	29.0%	29.9%	70.1%
LTR	29.2%	37.6%	33.2%	66.8%
3-Year Look-back				
RVM	43.4%	23.3%	33.3%	66.7%
FER	31.9%	40.0%	28.1%	71.9%
LTR	29.7%	41.1%	29.1%	70.9%
Modified 5-Year Look-back (10-Year Std. Dev.)				
RVM	41.2%	22.4%	36.4%	63.6%
FER	53.0%	19.9%	27.1%	72.9%
LTR	24.5%	40.5%	35.0%	65.0%
Modified 3-Year Look-back (10-Year Std. Dev.)				
RVM	39.2%	21.1%	39.7%	60.3%
FER	53.2%	20.1%	26.8%	73.2%
LTR	27.2%	39.1%	33.7%	66.3%

equities would beat bonds. Our t-tests would seem to indicate that the performance of the Modified 3-Year FER paradigm is more than just a happy coincidence, but the shallowness of its return inputs flies in the face of conventional wisdom. Because of this, we explored several contrarian return input schemes with different mean-reverting behaviors but were not able to uncover any that didn't result in a destruction of value over the testing period. In addition, quarterly optimization shifts were tried but with no notable improvement over the annual paradigms.

Higher Risk = Higher Return

Of course, one very good explanation of the better performance of the FER Optimized Portfolios vs. the Naive Allocation is the fact that it is the only paradigm that allowed a higher average equity allocation over the 21-year period. Table 5 shows the average allocation for each of the original paradigms, and FER consistently averaged 3% above the 70% Naive Allocation to equity. The obvious conclusion is that the MVPO value added is merely a reward for taking more equity risk; however, a second look at the significantly higher Sharpe Ratios for the FER paradigms in Table 2 reduces our confidence in this inference.

In order to dispel it fully, we examined the performance of a fixed blend allocation consisting of the average mix of the Modified 3-Year FER Constrained (i.e., 5% maximum shifts) paradigm. The performance analysis is shown in Table 6 with a reduced performance premium, but still a very compelling significance level well below 5% (Table 7).

The Emperor's New Clothes

What conclusions can be drawn from all of this analysis? First, and foremost, MVPO is an endeavor brimming with promise and opportunity. Unfortunately, this is too easily the opportunity to maximize input errors rather than performance. Our experiment suggests that recent experience is only valuable as it relates to correlations. Conversely, the intuitively appealing tendency to allow short-term risk and return manifestations to become inputs in MVPO paradigms results in portfolios that are inferior to a simple naive fixed blend. In actual practice, the extreme shifts that these short-term MVPO paradigms often dictate are never really implemented as practitioners restrain their optimizers with subjectively derived constraints to prevent these aggressive asset allocation shifts. As a result, the "optimum" portfolio becomes more a product of these subjective constraints than the MVPO inputs.

The fact that the experiment identified one set of MVPO paradigms that added value over the Naive Portfolio is encouraging; however, it is questionable whether this result would hold up under the burden of complexity necessary for it to be useful in the real world. The likelihood of fund overseers and consultants abdicating the rule of their own common sense to such a mathematical construction is practically zero. These performance margins, while statistically significant, are slim. Trustees and consultants will undoubtedly view

Table 6 - Annualized Premiums of 3-Year Modified FER Constrained Paradigm Using Naive Allocation Risk Over 53% S&P / 20% EAFE / 27% LB Agg

	2001	2000	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981
1981	0.58	0.53	0.55	0.54	0.57	0.64	0.66	0.59	0.66	0.80	0.80	0.87	1.03	1.21	1.26	1.16	0.38	0.21	0.15	0.35	0.18
1982	0.60	0.55	0.57	0.57	0.60	0.67	0.70	0.63	0.70	0.87	0.88	0.96	1.16	1.38	1.47	1.40	0.44	0.22	0.13	0.55	
1983	0.60	0.55	0.57	0.57	0.60	0.68	0.71	0.63	0.72	0.90	0.91	1.01	1.25	1.52	1.66	1.62	0.41	0.06	-0.28		
1984	0.65	0.60	0.63	0.62	0.66	0.75	0.79	0.71	0.81	1.03	1.06	1.20	1.51	1.89	2.15	2.27	0.76	0.37			
1985	0.66	0.61	0.64	0.64	0.68	0.79	0.83	0.75	0.87	1.11	1.16	1.34	1.76	2.31	2.82	3.41	1.22				
1986	0.63	0.58	0.61	0.60	0.64	0.75	0.80	0.70	0.83	1.10	1.15	1.36	1.88	2.62	3.52	5.51					
1987	0.35	0.27	0.28	0.24	0.26	0.34	0.34	0.19	0.25	0.46	0.39	0.47	0.79	1.35	1.80						
1988	0.25	0.15	0.15	0.10	0.10	0.17	0.16	-0.04	-0.01	0.19	0.03	0.02	0.23	0.87							
1989	0.21	0.09	0.08	0.02	0.02	0.09	0.06	-0.19	-0.18	0.03	-0.25	-0.37	-0.43								
1990	0.25	0.14	0.13	0.07	0.07	0.16	0.13	-0.14	-0.13	0.16	-0.16	-0.32									
1991	0.31	0.19	0.19	0.13	0.13	0.25	0.24	-0.09	-0.05	0.45	0.03										
1992	0.34	0.21	0.21	0.14	0.15	0.29	0.29	-0.13	-0.09	0.79											
1993	0.28	0.13	0.11	0.01	0.01	0.15	0.10	-0.61	-1.06												
1994	0.45	0.30	0.31	0.23	0.27	0.55	0.68	-0.20													
1995	0.55	0.39	0.43	0.36	0.46	1.00	1.79														
1996	0.37	0.14	0.12	-0.08	-0.15	0.30															
1997	0.39	0.10	0.06	-0.28	-0.61																
1998	0.60	0.32	0.39	0.07																	
1999	0.75	0.43	0.70																		
2000	0.76	0.21																			
2001	1.30																				

Table 7 - Returns and Statistics Resulting from 3-Year Look-back Constrained FER Paradigm Using Naive Allocation Risk

				Premiums Over Average Fixed Allocation			
	Annualized Return	Annual Std. Dev.	Sharpe Ratio	Annualized Premium	Quarterly Average	Quarterly Std. Dev.	Significance
Fixed Average FER Allocation	12.84%	11.48%	0.5387				
FER (with Naive Risk)	13.42%	11.90%	0.5682	0.58%	0.13%	0.53%	1.37%

any active allocation changes in light of the value added by shifts implemented over the most recent 12 to

24 months. If any temporary drought in premium should occur, it is probable that the paradigm will be altered or abandoned entirely, easily destroying any potential premium it might have generated.

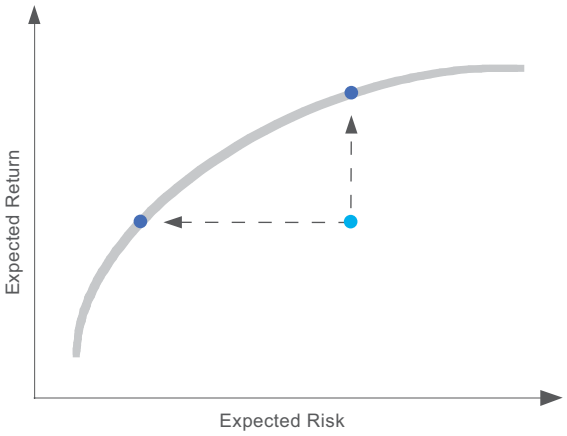
A Question of Risk

Although MVPO is far from a shaft of heavenly light illuminating the optimum portfolio, it still provides an excellent tool for the education of trust fiduciaries. The usefulness of MVPO in demonstrating the risk-reducing benefits of diversification to trustees who may be reticent to place funds into an unfamiliar or "risky" asset class cannot be overstated. The MVPO "efficient frontier" construct ultimately reduces asset allocation to its very essence: a risk/return dilemma. By relegating ourselves to portfolios on **some** frontier (efficient or not), expected return becomes merely a dependent variable leaving us to focus solely on the question of portfolio risk. And this is the very crux of the issue, for even if we had perfectly clairvoyant inputs, MVPO can never answer the most fundamental question: *"What is the appropriate level of 'risk' for the portfolio?"*

Answering this question requires the translation of risk from its usual abstruse statistical manifestation into expressions that are more meaningful to the typical Trustee: expressions such as the distribution of expected future contributions, accounting costs, or payouts and comparative point estimates like the expected Net Present Value of these same measures.

These convert “risk” from a four-letter word into tangible concepts that allow trustees to make informed decisions regarding asset allocation as it relates to the costs and liabilities the assets were intended to service.

Figure 7 - Efficient Frontier



Conclusions

The obvious conclusion is that cavalierly derived inputs can easily render the entire MVPO process useless or, even worse, destructive. Our results indicate the only MVPO paradigms that generated statistically significant performance margins focused almost entirely on the risk posture of the considered assets with relatively informationless return expectations. Whether these margins could withstand the complexity and the imposition of individual biases that are all but inevitable in real world applications is questionable.

In the absence of return predictions, the value of MVPO lies not in its illusory ability to create superior portfolio allocations, but rather as an aid in exploring the impact of various portfolio risk postures. This exploration can only take place if risk can be expressed in terms that are meaningful instead of vague statistical constructs. Anything less treats portfolio assets as if they are an end unto themselves rather than existing for a purpose: offsetting real liabilities and costs.

The past 15 years have seen MVPO grow from an obscure academic argument for diversification to a ubiquitous elixir to cure what ails the portfolio of every individual investor. Analysis indicates that its actual effect lies somewhere between harmless placebo and toxic process depending on the skill with which it is applied. *Caveat emptor.*

Notes

Significance tests assume that the return differences (premiums) are normally distributed with mean ❍. Thus $\overline{Z}$ if is the sample mean and S is the sample variance for n quarterly premiums, then T_{n-1} has a t-distribution with $n\text{-}1$ degrees of freedom where,

$$T_{n-1} = \frac{\left(\overline{Z} - \mu\right)\sqrt{n}}{S}$$

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Chopra, V. K., and William T. Ziemba. “The Effect of Errors in Means, Variances and Covariances on Optimal Portfolio Choice.” Journal of Portfolio Management, Summer 1993. pp 51-62.

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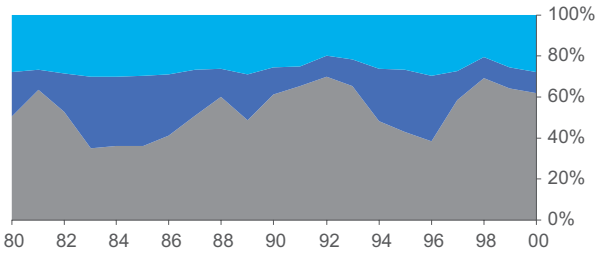
Appendix

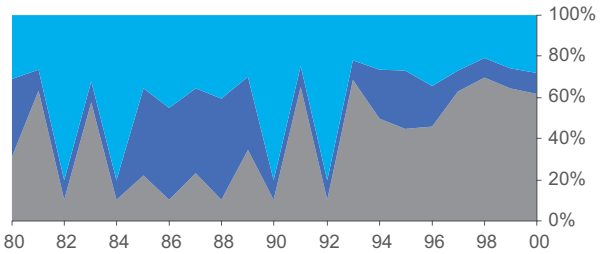
Asset Allocation Paths for 3-Year Look-back

Naive Allocation Risk

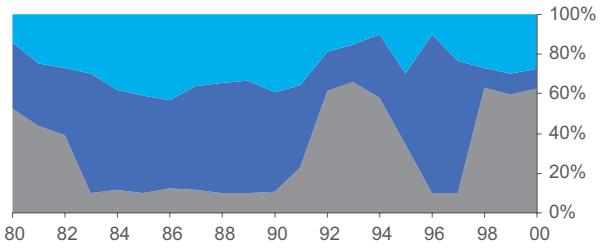
3-Year Modified FER Optimization



3-Year Modified RVM Optimization



3-Year Fixed Expected Return Optimization



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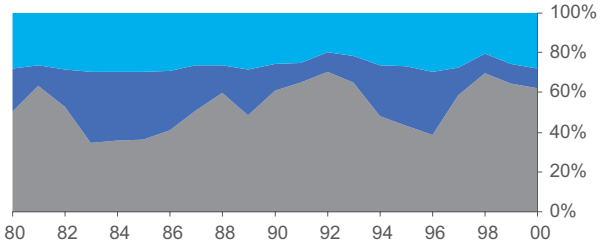
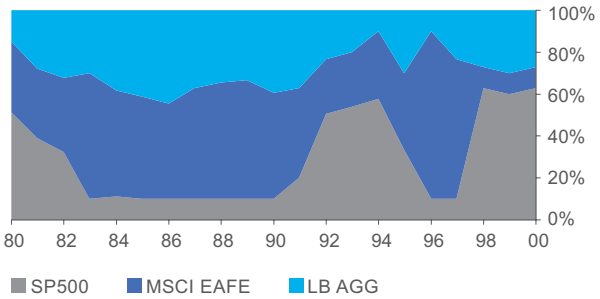
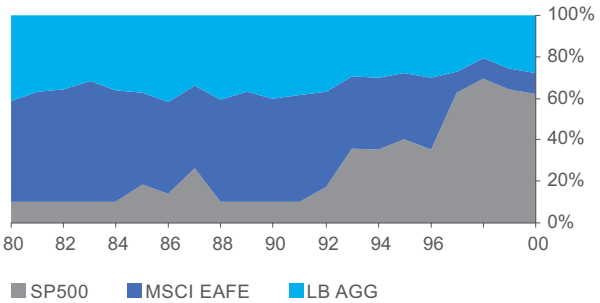


3-Year Long-Term Returns Optimization



Modified 3-Year LTR Optimization

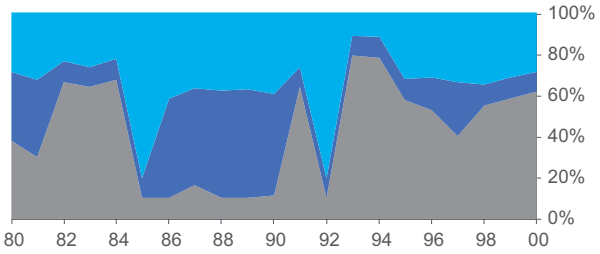


Appendix

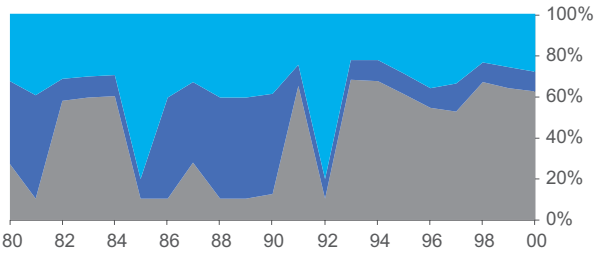
Asset Allocation Paths for 5-Year Look-back

Naive Allocation Risk

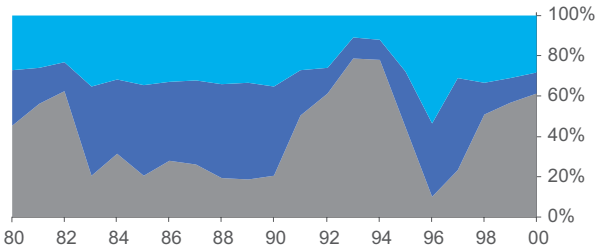
5-Year Rear-View Mirror Optimization



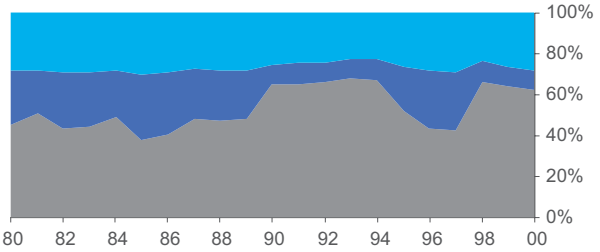
Modified 5-Year RVM Optimization



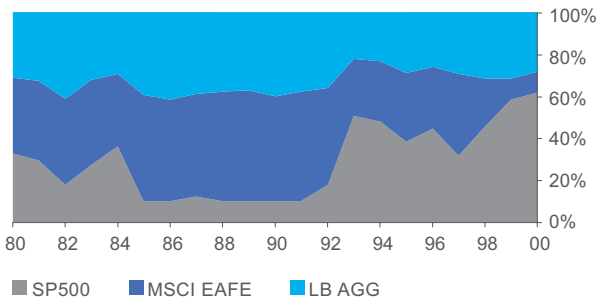
5-Year Fixed Expected Return Optimization



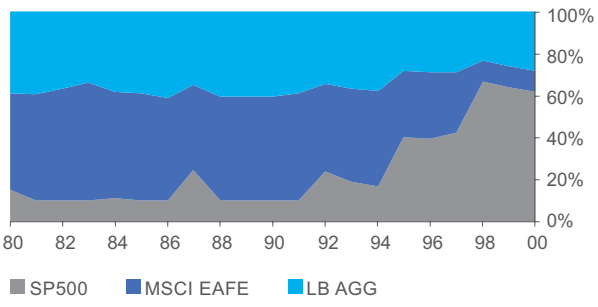
Modified 5-Year FER Optimization



5-Year Long-Term Returns Optimization



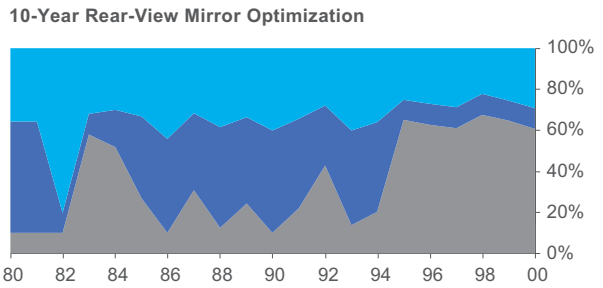
Modified 5-Year LTR Optimization

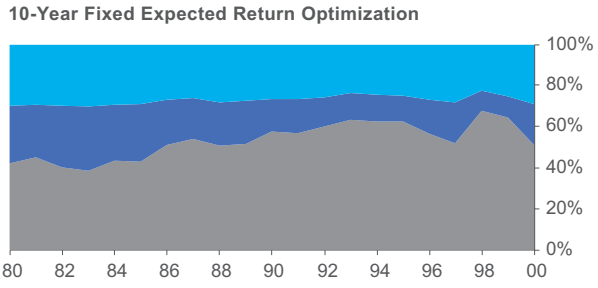


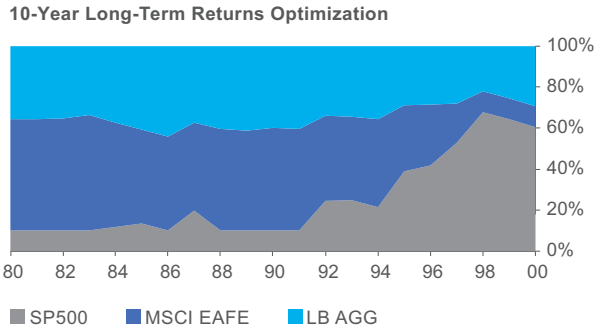
Appendix

Asset Allocation Paths for 10-year Look-back

Naive Portfolio Risk



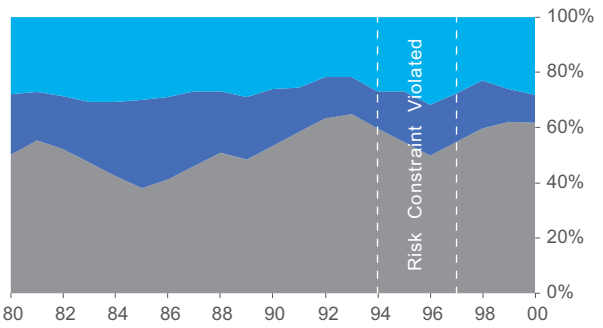




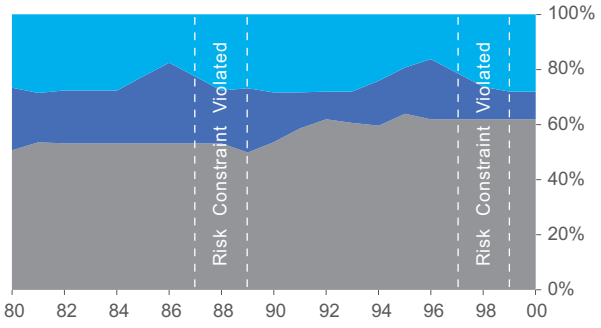
Appendix

Asset Allocation Paths for Constrained Shift Optimizations

Naive 3-Year Modified FER Constrained Shift Optimization



Prescient 3-Year Modified FER Constrained Shift Optimization



About the Author

Randall Doser joined Compass Advisors as Vice President in March of 2001. A Fellow of the Society of Actuaries, an Enrolled Actuary (inactive) and a Member of the American Academy of Actuaries, he has a strong background in the mathematics of finance and extensive experience in the legal and accounting aspects of benefit plan liabilities and assets. Most recently, he was the Manager of Benefit Investments for Freightliner LLC, overseeing the company's $900 million portfolio of pension, 401 (k) and retiree medical assets since 1998. Before that he spent three years with the consulting firm Watson Wyatt Worldwide in their Portland and Minneapolis offices doing retiree medical and defined benefit retirement plan valuation and design for employers ranging in size from 12 to 60,000+ employees. Prior to 1995 he was an associate consultant with Howard Johnson & Company in Portland, Oregon doing defined contribution and defined benefit retirement plan design, administration and valuation for small to mid size companies. Randall graduated in 1990 Magna Cum Laude from Oregon State University with a B.S. in Mathematical Sciences.

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